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FOR THE COMMANDER

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PROPERTIES OF THE LINEAR POLARIZATION BISTATIC SCATTERING MATRIX

G. E. HEATH

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ABSTRACT

When the plane containing the transmitter and receiver lines of sight is used as the polarization reference plane, the scattering matrix characterizing linearly polarized bistatic scattering from plane-symmetric targets exhibits scattering geometry symmetries which can be exploited to reduce the time and expense in obtaining bistatic scattering data from static range measurements and theoretical or computer calculations.

In this report the linear polarization symmetry relations are derived and used to determine the minimum number of measurements needed for the complete characterization of target scattering.

CONTENTS

I. INTRODUCTION

The purpose of this paper is to document the symmetry properties of the Linear Polarization Bistatic Scattering Matrix and to determine the minimum number of measurements for its complete specification. Properties of the Circular Polarization Bistatic Scattering Matrix have been considered in Ref. 1.

The Linear Polarization Bistatic Scattering Matrix $S = \{a_{ij}\}$ is defined by the relation

$$\begin{bmatrix} E_H{}^s \\ E_V{}^s \end{bmatrix} = \begin{bmatrix} a_{HH} & a_{HV} \\ a_{VH} & a_{VV} \end{bmatrix} \begin{bmatrix} E_H{}^i \\ E_V{}^i \end{bmatrix} \quad . \tag{1}$$

$E_H{}^i$ and $E_V{}^i$ are the components of the incident electric field in the directions of unit polarization vectors $\hat{H}_i$ and $\hat{V}_i$ that are perpendicular to each other and the unit incident wave vector $\hat{k}_i$. $\hat{H}_i$ and $\hat{V}_i$ are defined to be parallel and perpendicular, respectively, to a transmitter polarization reference plane which contains $\hat{k}_i$. Similarly, $E_H{}^s$ and $E_V{}^s$ are the components of the scattered electric field in the directions of unit polarization vectors $\hat{H}_s$ and $\hat{V}_s$ that form a mutually perpendicular triplet with the unit scattered wave vector $\hat{k}_s$. $\hat{H}_s$ and $\hat{V}_s$ are parallel and perpendicular, respectively, to a receiver polarization reference plane which contains $\hat{k}_s$. The following analysis is restricted to bistatic polarization conventions that reduce to $\hat{H}_s = \hat{H}_i$ and $\hat{V}_s = \hat{V}_i$ when $\hat{k}_s = -\hat{k}_i$ (monostatic scattering). However, when $\hat{k}_s \neq -\hat{k}_i$ (bistatic scattering) the two polarization reference planes are, in general, different. For example, the θ and φ polarizations used in Ref. 1

correspond to the H and V polarizations, respectively, illustrated in Fig. 1. In this case both polarization reference planes are defined to contain the z axis.

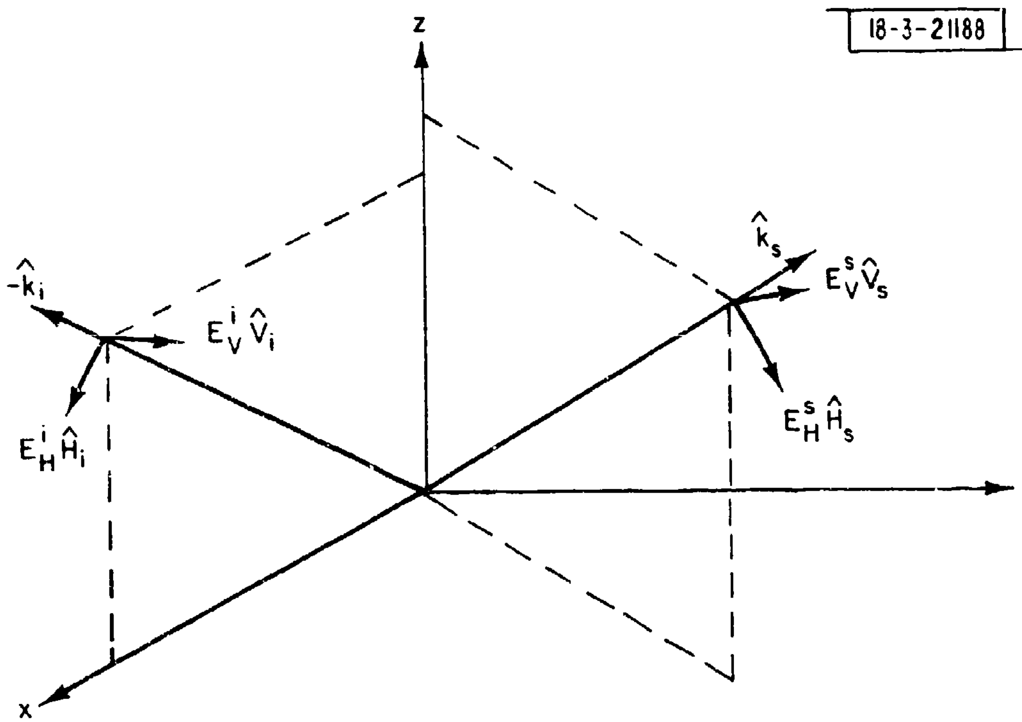

Fig. 1. Horizontal and vertical polarization components of the incident and scattered electric fields.

II. CONDITION FOR A SYMMETRIC SCATTERING MATRIX

For a given target and radar frequency, S only depends on $\hat{k}_i$ and $\hat{k}_s$. In the general case of bistatic scattering S is not symmetric, i.e.,

$$a_{VH}(\hat{k}_i, \hat{k}_s) \not\equiv a_{HV}(\hat{k}_i, \hat{k}_s) \qquad \hat{k}_s \neq -\hat{k}_i \tag{2}$$

where the symbol $\not\equiv$ is used to denote "not identically equal to." Therefore eight independent measurements (four of cross section, four of phase) are generally necessary to specify S for a given scattering geometry [Ref. 2, p. 18]. The inequality in Eq. (2) does not violate the principle of electromagnetic reciprocity which can be written in the form [Ref. 3, p. 252]

$$a_{HH}(-\hat{k}_s, -\hat{k}_i) = a_{HH}(\hat{k}_i, \hat{k}_s)$$

$$a_{VV}(-\hat{k}_s, -\hat{k}_i) = a_{VV}(\hat{k}_i, \hat{k}_s)$$

and (3)

$$a_{VH}(-\hat{k}_s, -\hat{k}_i) = a_{HV}(\hat{k}_i, \hat{k}_s) \quad .$$

The statement that S is always symmetric when the target is a perfect conductor [Ref. 4, p. 27-4] is, in general, erroneous. However, it is not difficult to show that S will always be symmetric when both of the following conditions are satisfied:

a) The bistatic plane (containing $\hat{k}_i$ and $\hat{k}_s$) is a symmetry plane of the target.

b) The transmitter and receiver polarization reference planes intersect the bistatic plane at the same angle.

The proof in the following paragraph follows from the result that S is diagonal ($a_{vh} = a_{hv} = 0$) when the bistatic plane is both [Ref. 1]

a) a symmetry plane of the target

and

b) the polarization reference plane for both incident and scattered electric fields.

(In the notation of Ref. 1, $a_{\varphi\theta}(y) = a_{\theta\varphi}(y) = 0$ when $y = 0$.)

When the transmitter and receiver polarization reference planes are both rotated from the bistatic plane by the angle ξ, the relationships between the field components in the old and new polarization reference systems are given by standard rotation of coordinate system formulae. The resulting transformation for the components of S is given by

$$\begin{bmatrix} a_{HH} & a_{HV} \\ a_{VH} & a_{VV} \end{bmatrix} = \begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix} \begin{bmatrix} a_{hh} & a_{hv} \\ a_{vh} & a_{vv} \end{bmatrix} \begin{bmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{bmatrix} \tag{4}$$

The result that $a_{VH} = a_{HV}$ when $a_{vh} = a_{hv} = 0$ follows from Eq. 4 by matrix multiplication.

III. SYMMETRY RELATIONS FOR BODIES OF REVOLUTION

When the bistatic plane is the polarization reference plane for both incident and scattered fields, the Linear Polarization Scattering Matrix coefficients for scattering from bodies of revolution exhibit symmetries which can be exploited to reduce the minimum number of measurements necessary to specify S. As shown in the following paragraph, the linear polarization symmetry properties are readily obtained from the circular polarization symmetry properties given in Table 5 of Ref. 1.

Linear polarization unit vectors for the Bistatic Measurement System described in Ref. 1 are shown in Fig. 2. The various quantities in Fig. 2 are defined in Table 1. Circular polarization field components for exp $(i\omega t)$ time variations can be defined by

$$\begin{bmatrix} E_H{}^s \\ E_V{}^s \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ -i & i \end{bmatrix} \begin{bmatrix} E_R{}^s \\ E_L{}^s \end{bmatrix} \tag{5}$$

and

$$\begin{bmatrix} E_R{}^i \\ E_L{}^i \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} E_H{}^i \\ E_V{}^i \end{bmatrix} \tag{6}$$

where

$$\begin{bmatrix} E_R{}^s \\ E_L{}^s \end{bmatrix} = \begin{bmatrix} a_{RR} & a_{RL} \\ a_{LR} & a_{LL} \end{bmatrix} \begin{bmatrix} E_R{}^i \\ E_L{}^i \end{bmatrix} . \tag{7}$$

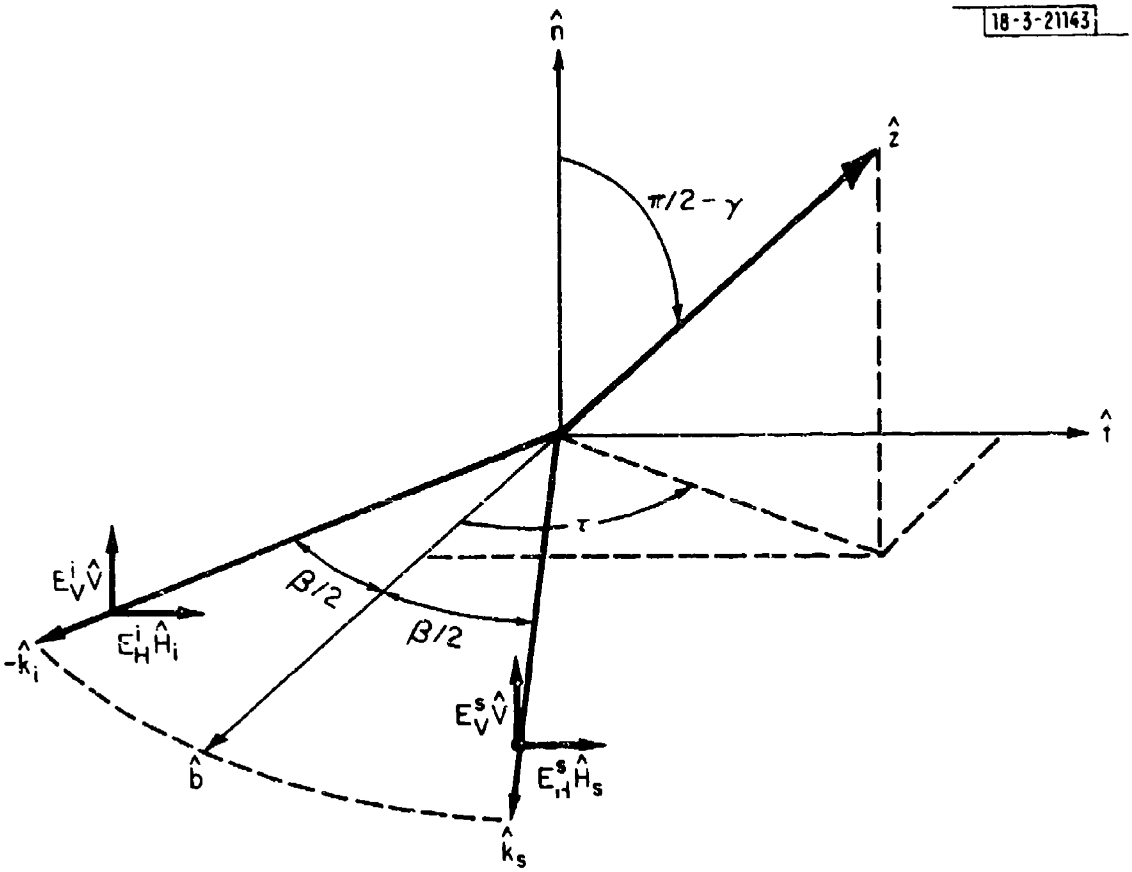

Fig. 2. Horizontal and vertical polarization for the bistatic measurement system.

TABLE 1

BISTATIC MEASUREMENT COORDINATES (τ, β, γ)

$\hat{z}$		Forward Directed Principal Axis Unit Vector of Target	
$\hat{k}_i$		Unit Incident Wave Vector	
$\hat{k}_s$		Unit Scattered Wave Vector	
$\hat{b}$	=	$(\hat{k}_s - \hat{k}_i)/\lvert\hat{k}_s - \hat{k}_i\rvert$	(Bistatic Unit Bisector)
$\hat{t}$	=	$(\hat{k}_s + \hat{k}_i)/\lvert\hat{k}_s + \hat{k}_i\rvert$	(Bistatic Unit Tangent)
$\hat{n}$	=	$\hat{b} \times \hat{t}$	(Bistatic Unit Normal)
τ		Bistatic Turntable Angle	$(-\pi < \tau \leq \pi)$
β		Bistatic Angle	$(0 \leq \beta \leq \pi)$
γ		Bistatic Pitch Angle	$(-\pi/2 \leq \gamma \leq \pi/2)$
$\hat{V} = \hat{n}$		Vertical Polarization Unit Vector	
$\hat{H}_i$		Incident Wave Horizontal Polarization Unit Vector	
$\hat{H}_s$		Scattered Wave Horizontal Polarization Unit Vector	
E_V^i		Incident Field Vertical Polarization Component	
E_H^i		Incident Field Horizontal Polarization Component	
E_V^s		Scattered Field Vertical Polarization Component	
E_H^s		Scattered Field Horizontal Polarization Component	

Combining Eqs. 1 and 5-7 yields

$$a_{HH} = \frac{1}{2} \{a_{LR} + a_{RL} + a_{RR} + a_{LL}\} \quad ,$$

$$a_{VV} = \frac{1}{2} \{a_{LR} + a_{F} - a_{RR} - a_{LL}\} \quad ,$$

$$a_{HV} = \frac{-i}{2} \{a_{LR} - a \quad + a_{RR} - a_{LL}\} \quad ,$$

and (8)

$$a_{VH} = \frac{i}{2} \{a_{LR} - a_{RL} - a_{RR} + a_{LL}\} \quad .$$

The circular polarization scattering coefficients in Eq. 8 differ from those in Ref. 1 by phase shifts that vary with scattering geometry. Nevertheless, they still satisfy the symmetry relations in Table 5 of Ref. 1. Combining Eq. 8 and the circular polarization symmetry relations of Ref. 1 yields the linear polarization symmetry relations in Table 2.

TABLE 2

LINEAR POLARIZATION SYMMETRY RELATIONS FOR A BODY OF REVOLUTION
(Bistatic Measurement Coordinates)

Electromagnetic Reciprocity

$$a_{HH}(-\tau, \beta, -\gamma) = a_{HH}(\tau, \beta, \gamma)$$

$$a_{VV}(-\tau, \beta, -\gamma) = a_{VV}(\tau, \beta, \gamma)$$

$$a_{VH}(-\tau, \beta, -\gamma) = a_{HV}(\tau, \beta, \gamma)$$

Pitch Angle Reflection

$$a_{HH}(\tau, \beta, -\gamma) = a_{HH}(\tau, \beta, \gamma)$$

$$a_{VV}(\tau, \beta, -\gamma) = a_{VV}(\tau, \beta, \gamma)$$

$$a_{HV}(\tau, \beta, -\gamma) = -a_{HV}(\tau, \beta, \gamma)$$

$$a_{VH}(\tau, \beta, -\gamma) = -a_{VH}(\tau, \beta, \gamma)$$

Turntable Angle Reflection

$$a_{HH}(-\tau, \beta, \gamma) = a_{HH}(\tau, \beta, \gamma)$$

$$a_{VV}(-\tau, \beta, \gamma) = a_{VV}(\tau, \beta, \gamma)$$

$$a_{VH}(-\tau, \beta, \gamma) = -a_{HV}(\tau, \beta, \gamma)$$

IV. INDEPENDENT LINEAR POLARIZATION MEASUREMENTS

As stated in Section II, eight independent measurements are generally necessary to specify S for a given radar frequency and scattering geometry. However, when the bistatic plane is the polarization reference plane for both incident and scattered fields, the linear polarization static range measurement patterns for scattering from bodies of revolution exhibit the symmetries in Table 2. As explained in the following paragraph, the static pattern symmetries can be exploited to reduce the number of measurements needed for the complete characterization of the bistatic scattering properties of the target.

Experimental bistatic cross section and phase static patterns are typically measured over a 360° interval in turntable angle ($-180^\circ < \tau \leq 180^\circ$) with pitch angle γ and bistatic angle β ($\beta \neq 0$) held fixed [Ref. 5]. Since the linear polarization fields are either symmetric or antisymmetric in γ, it is sufficient to make only non-negative (or non-positive) pitch measurements. The cross polarized fields VH and HV are antisymmetric in γ. Therefore, for fixed β, it is sufficient to make only four measurements when $\gamma = 0$: the cross sections and phases of the principal polarization fields (HH and VV). When $\gamma \neq 0$ VH patterns can be obtained by reflecting the HV patterns about $\tau = 0$ (or vice versa). Therefore, for fixed β and $\gamma(\gamma \neq 0)$ only six measurements are necessary. Consequently, for fixed $\beta(\beta \neq 0)$, three sets of 360° static patterns (e.g., HH and VV for $0 \leq \gamma \leq 90^\circ$ and HV for $0 < \gamma \leq 90^\circ$) are sufficient for the complete characterization of scattering from a body of revolution. However, since the HH and VV 360° patterns are symmetric about $\tau = 0$, measurement redundancy is not completely eliminated.

The measurement redundancy inherent in 360° experimental static range patterns can be avoided when making theoretical or computer calculations. For fixed β four sets of 180° ($0 \leq \tau \leq 180^{\circ}$) static patterns (e.g., HH and VV for $0 \leq \gamma \leq 90^{\circ}$; HV and VH for $0 < \gamma \leq 90^{\circ}$) are sufficient.

For a target whose physical shape is a body of revolution but whose electrical properties vary in roll angle ρ (modulo 2π) about the physical symmetry axis, Table 2 is not applicable. Consequently, unless the target has roll angle symmetries, all eight static patterns over a 360° range of roll angles are needed for the complete characterization of the target when β and $|\gamma|$ are fixed. Measurement redundancy can be avoided by using the reciprocity relations of Eq. 2 in the form

$$a_{ji}(-\tau,\ \beta,\ -\gamma,\ \rho \pm 180^{\circ}) = a_{ij}(\tau,\ \beta,\ \gamma,\ \rho) \tag{9}$$

to obtain negative pitch patterns from positive pitch patterns (or vice versa).

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